POWER SYSTEM STABILITY CONTINUED...

A power system is stable if when disturbed from a state of equilibrium, it returns to the state of equilibrium (same state)

Essentially all synch-machines in the system must be operating in a steady state and are in synchronism and operating at the same freq.

If for some reason one machine is accelerated, restoring tourques are developed decelerating the fast machine and accelerating others until the freq difference disapears. If restoring tourques are insufficient, sustained freq (or speed) difference appears, which results in beat freq oscilations of currents, voltages, active/reactive powers.

When this happens, circuit breakers will opperate to shut down vital motors etc. and general breakdown of a system will take place.

System shut down causes considerable delays and can be very costly. Freq variations are often no more then one to three percent of the nominal freq.

THE SWING EQUATION

The dynamic equation relating the internal tourque to the net accelerating tourque of syncronous machine rotor, is called the "swing equation" and is given as:

$$\boxed{J \frac{d^2\theta}{dt^2} = T_a \qquad (N \cdot m)}$$

mech. driving tourque

$T_a = T_m - T_e$

internal tourque which is the product of inertia of all rotating mass attached to the shaft of the machine and angular acceleration.

load electrical tourque.

The angular position of the rotor Θ may be expressed as the sum of angles.

$$\boxed{\Theta = \alpha + \omega_r t + \delta}$$

α : the constant angle that is needed if the angle δ is measured from an axis different from the angular reffrence.

δ : the time varing angle, and represents deviations from the related angular displacement.

<u>NOW</u>

$$J \frac{d^2\Theta}{dt^2} = T_m - T_e$$

$$\Theta = \alpha + \omega_r t + \delta$$

$$\frac{d\Theta}{dt} = \omega_r + \frac{d\delta}{dt}$$

$$\frac{d^2\Theta}{dt^2} = \frac{d^2\delta}{dt^2} = \ddot{\delta}$$

$$\therefore \quad \boxed{J\ddot{\delta} = T_m - T_e}$$

(*) can be written in terms of powers, by multiplying by ω

$$J\omega\ddot{\delta} = P_m - P_e$$

$J\omega = M$: inertia constant which is angular momentum.

Another form is known as the normalized form which is obtained by dividing (*) through by T_r (rated tourque)

$$\frac{J}{T_r}\ddot{\delta} = \frac{T_m}{T_r} - \frac{T_e}{T_r}$$

The right hand side of the above equation can be further manipulated to yeild a frequently used form, by introducing K.E.

$$KE = \frac{1}{2} J \omega_r^2 = W_K$$

$$\frac{J}{T_r} = \frac{2 W_K}{\omega_r^2 T_r}$$

but

$$P_R = \omega_r T_r$$

$$\therefore \quad \frac{J}{T_r} = \frac{2 W_K}{\omega_r P_r}$$

hence

$$\frac{2 W_K}{\omega_r P_r}\ddot{\delta} = \frac{T_m}{T_R} - \frac{T_e}{T_R}$$

A constant that has proved useful is defined by:

$$H = \frac{W_K}{P_R}$$

by definition the units of H are seconds as a result, we may write our swing equation as

$$\frac{2H}{\omega_r}\ddot{\delta} = T_{m_{P.U.}} - T_{e_{P.U.}}$$

we may write the above as

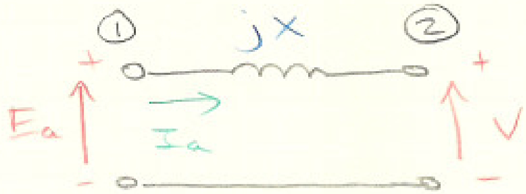
$$\frac{2H}{\omega_R}\ddot{\delta} = P_{m_{p.u.}} - P_{e_{p.u.}}$$

where

$$T_{p.u.} = P_{p.u.}$$

THE ELECTRICAL POWER EQUATION:

Recall from synch-machine theory neglecting R_a (armature resistance) then



Neglecting R_a

$$P_1 = P_2 = \frac{|E||\bar{V}|}{X}\sin\delta$$ per phase

$$Q_1 = \frac{|E|^2 - |E||\bar{V}|\cos\delta}{X}$$

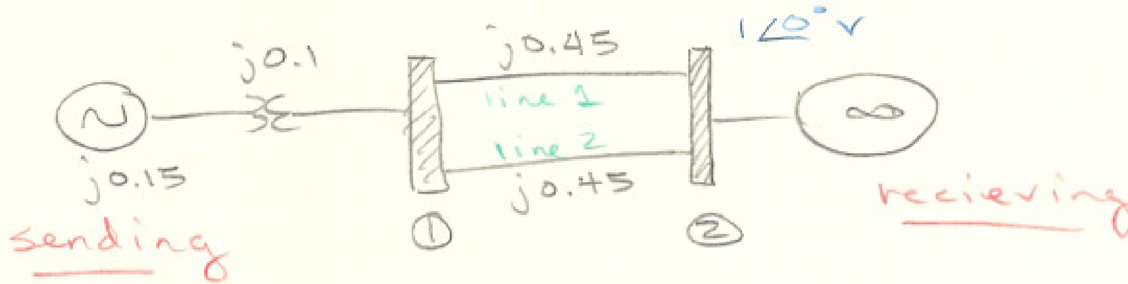
$$Q_2 = \frac{|E||V|\cos\delta - |V|^2}{X}$$

where X is the total reactance.

EX: A synch-machine is connected to an infinite bus through a transformer and a double-cct transmission line as shown below.

Assume $\bar{V}_\infty = 1\angle 0^\circ$

Find P_e = electrical power in terms of $\bar{E}$



Total $\bar{Z} = j0.15 + j0.1 + j0.45 \,//\, j0.45$

$$\bar{Z} = j0.475$$

$$X = |\bar{Z}| = 0.475$$

$$|\bar{V}| = 1.0$$

Then

$$P_e = \frac{|E|(1)\sin\delta}{0.475} = 2.1053\,|E|\sin\delta$$